# Tire tracks geometry, Menzin's conjecture, continuous and discrete bicycle transformation, and complete integrability

Sergey Tabachnikov
(Pennsylvania State University)

This talk concerns a simple model of bicycle motion: a bicycle is a segment of fixed length that can move in the plane so that the velocity of the rear end is always aligned with the segment. The trajectory of the front wheel and the initial position of the bicycle uniquely determine its motion and its terminal position; the monodromy map sending the initial position to the terminal one arises. This circle mapping is a Moebius transformation, a remarkable fact that has various geometrical and dynamical consequences. Moebius transformations belong to one of the three types: elliptic, parabolic and hyperbolic. I shall outline a proof of a 100 years old conjecture: if the front wheel track is an oval with area at least Pi then the respective monodromy is hyperbolic.

The rear wheel track and a choice of direction determine the front wheel track; changing the direction to the opposite, yields another front track. The two front tracks are related by the bicycle, or Backlund-Darboux, transformation which defines a discrete time dynamical system on the space of curves. This system is completely integrable and closely related with a well studied completely integrable continuous time dynamical system, the filament (or binormal, or smoke ring) equation. There is also a discrete version of the Backlund-Darboux transformation, acting on polygons, rather than smooth curves.