Multi-dimensional Markov chains with special structures of generators and their application in natural sciences

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Markov chains have found a lot of versatile applications for modelling various real world objects. Important particular example of continuous time Markov chain is so called birth and death process. The generator of a birth and death process is a three-diagonal matrix. From the early beginning, birth and death processes were applied for modelling the size of population of insects (this expains the name of such processes) in various biological studies. Currently, they are popular for modelling social systems (immigration and emigration), demography, ecology, genetics, molecular evolution, queue lengths in many physical, technical, economical and other systems. Evident shortcoming of application of a birth and death process to description of some population is an assumption that, during his/her life, each member of population can die or give a life to a child at a constant rate. Sure, in real life systems the birth and death rates essentially depend on the sex and age of the members of population. So, to have a more adequate model of dynamics of population, it is necessary to keep additionally track of this information about the sex and age. In such a way, the dynamics of population is described not only by its size (is is called sometimes as a level of a process), but also by the final set of some additional features. Thus, the process describing the dynamics of population is a multi-dimensional one. Under some proper enumeration of the components of this process, its generator has a three-block-diagonal structure. Such process is called in literature as quasi-birth-and-death process. The theory of quasi-birth-and-death processes with Toeplitz-like type structure of a generator and its generalizations to the cases of block-upper-Hessenberg and block-lower-Hessenberg Toeplitz-like structure of generator was developed by M. Neuts.

In this talk, well-known results by M. Neuts are shortly described and the original results relating to investigation of two particular structures of a generator of Markov chain are presented. One structure is block-upper-Hessenbergian with approaching to Toeplitz-like type when the level increases. Markov chains having such a structure of generator well fit for

description of real world processes with the birth and death rates proportional to the number of a level, e.g., for description of queues with customers retrial and queues with impartient customers. The second structure is a combination of block-upper-Hessenberg Toeplitz-like structure and structure with equal block rows of a generator. Such Markov chains are suitable, e.g., for description of queueing systems with exhaustive group service or with disasters. Ergodicity conditions are presented. Numerically stable algorithms for computation of stationary distribution are derived.

Literature:

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