

THE INFINITY COMPUTER AND NUMERICAL COMPUTATIONS WITH INFINITIES AND INFINITESIMALS

Yaroslav D. Sergeyev, Ph.D., D.Sc., D.H.C.

Distinguished Professor and Head of Numerical Calculus Laboratory

University of Calabria, Rende (CS), Italy

Professor, N.I. Lobatchevsky University of Nizhni Novgorod, Russia

<http://si.deis.unical.it/~yaro/cv.html>

Abstract

The lecture introduces a new methodology allowing one to execute numerical computations with finite, infinite, and infinitesimal numbers (see [1-14]) on a new type of a computer – the Infinity Computer (see EU, USA, and Russian patents [2]). The new approach is based on the principle ‘The part is less than the whole’ introduced by Ancient Greeks that is applied to all numbers (finite, infinite, and infinitesimal) and to all sets and processes (finite and infinite). It is shown that it becomes possible to write down finite, infinite, and infinitesimal numbers by a finite number of symbols as particular cases of a unique framework different from that of the non-standard analysis.

The new methodology (see surveys [6,10]) evolves ideas of Cantor and Levi-Civita in a more applied way and introduces new infinite integers that possess both cardinal and ordinal properties as usual finite numbers (its relations with traditional approaches are discussed in [11,16]). It gives the possibility to execute computations of a new type and simplifies fields of mathematics where the usage of the infinity and/or infinitesimals is necessary (e.g., divergent series, limits, derivatives, integrals, measure theory, probability theory, fractals, etc., see [3-24]). Numerous examples and applications are given. A number of results related to the First Hilbert Problem are established.

In the following there are listed both operations that the Infinity Computer can execute and traditional computers are not able to perform and some of new areas of applications. The new approach allows:

- to substitute symbols $+\infty$ and $-\infty$ by sets of positive and negative infinite numbers, to represent them in the memory of the Infinity Computer and to execute arithmetical operations with all of them numerically, as we are used to do with usual finite numbers on traditional computers;
- to substitute qualitative descriptions of the type ‘a number tends to zero’ by precise infinitesimal numbers, to represent them in the memory of the Infinity Computer, and to execute arithmetical operations with them numerically as we are used to do with usual finite numbers using traditional computers;
- to calculate divergent limits, series, and improper integrals, providing as results explicitly written different infinite numbers, to be possibly used in further calculations on the Infinity Computer;
- to avoid appearance of indeterminate forms (e.g., in situations where it becomes necessary to calculate difference of two divergent series);
- to evaluate functions and their derivatives at infinitesimal, finite, and infinite points (infinite and infinitesimal values of functions and their derivatives can be also explicitly calculated);
- to study divergent processes at different infinite points;
- to introduce notions of lengths, areas, and volumes of fractal objects obtained after infinite numbers of steps and compatible with traditional lengths, areas, and volumes of non-fractal objects and to calculate all of them in a unique framework.

The Infinity Calculator using the Infinity Computer technology is presented during the talk. Additional information can be downloaded from the page <http://www.theinfinitycomputer.com>

Recent awards and distinctions

- Degree of Honorary Doctor, V.M. Glushkov Institute of Cybernetics of The National Academy of Sciences of Ukraine, Kiev, 2013
- Laureate of Applied Development and Research Competition in the field of Computer Technology “Computing Continuum: from Idea to Implementation” for the project “High accuracy computations on the Infinity Computer”, Intel Corporation and “Skolkovo” Innovation Center Development Fund, 2012
- Pythagoras International Prize for Mathematics (Premio Internazionale Pitagora per la Matematica), Crotone, Italy, 2010
- Outstanding Achievement Award from the 2010 World Congress in Computer Science, Computer Engineering, and Applied Computing, USA, 2010
- Lagrange Lecture, awarded jointly by Turin University and Polytechnic University of Turin, 2010
- Honorary plate from the Engineering Faculty of the Dokuz Eylul University, Izmir, Turkey, 2009
- MAIK “Nauka/Interperiodica” Prize for the best scientific monograph published in 2008 awarded by the Russian Academy of Sciences and Pleiades Publishing, Inc. (USA)
- Keynote address and honorary plaque at the general plenary session of the world multi-conference WMSCI 2008, Orlando, Florida, 2008

Selected references

1. Sergeyev Ya.D., Arithmetic of Infinity, Edizioni Orizzonti Meridionali, CS, 2003.
2. Sergeyev Ya.D. Computer system for storing infinite, infinitesimal, and finite quantities and executing arithmetical operations with them, EU patent 1728149, issued 03.06.2009; RF patent 2395111, issued 20.07.2010; USA patent 7,860,914 issued 28.12.2010.
3. Sergeyev Ya.D. (2007) Blinking fractals and their quantitative analysis using infinite and infinitesimal numbers, *Chaos, Solitons & Fractals*, 33(1), pp. 50-75.
4. Sergeyev Ya.D. (2008) A new applied approach for executing computations with infinite and infinitesimal quantities, *Informatica*, 19(4), 567-596.
5. Sergeyev Ya.D. (2009) Numerical computations and mathematical modelling with infinite and infinitesimal numbers, *Journal of Applied Mathematics and Computing*, 29, 177-195.
6. Sergeyev Ya.D. (2009) Evaluating the exact infinitesimal values of area of Sierpinski's carpet and volume of Menger's sponge, *Chaos, Solitons & Fractals*, 42, 3042–3046.
7. Sergeyev Ya.D. (2009) Numerical point of view on Calculus for functions assuming finite, infinite, and infinitesimal values over finite, infinite, and infinitesimal domains, *Nonlinear Analysis Series A: Theory, Methods & Applications*, 71(12), e1688-e1707.
8. Sergeyev Ya.D. (2010) Counting systems and the First Hilbert problem, *Nonlinear Analysis Series A: Theory, Methods & Applications*, 72(3-4), 1701-1708.
9. Sergeyev Ya.D., Garro A. (2010) Observability of Turing Machines: a refinement of the theory of computation, *Informatica*, 21(3), 425–454.
10. Sergeyev Ya.D. (2010) Lagrange Lecture: Methodology of numerical computations with infinities and infinitesimals, *Rendiconti del Seminario Matematico dell'Università e del Politecnico di Torino*, 68(2), 95–113.

11. Margenstern M. (2011) Using Grossone to count the number of elements of infinite sets and the connection with bijections, *p-Adic Numbers, Ultrametric Analysis and Applications*, 3(3), 196-204.
12. Sergeyev Ya.D. (2011) Higher order numerical differentiation on the Infinity Computer, *Optimization Letters*, 5, 575–585.
13. Sergeyev Ya.D. (2011) On accuracy of mathematical languages used to deal with the Riemann zeta function and the Dirichlet eta function, *p-Adic Numbers, Ultrametric Analysis and Applications*, 3(2), 129-148.
14. Sergeyev Ya.D. (2011) Using blinking fractals for mathematical modeling of processes of growth in biological systems, *Informatica*, 22(4), 559-576.
15. Iudin D.I., Sergeyev Ya.D., Hayakawa M. (2012) Interpretation of percolation in terms of infinity computations, *Applied Mathematics and Computation*, 218(16), 8099-8111.
16. Lolli G. (2012) Infinitesimals and infinities in the history of Mathematics: A brief survey, *Applied Mathematics and Computation*, 218(16), 7979-7988.
17. D’Alotto L. (2012) Cellular automata using infinite computations, *Applied Mathematics and Computation*, 218(16), 8077-8082.
18. De Cosmis S., De Leone R. (2012) The use of Grossone in Mathematical Programming and Operations Research, *Applied Mathematics and Computation*, 218(16), 8029-8038.
19. Margenstern M. (2012) An application of Grossone to the study of a family of tilings of the hyperbolic plane, *Applied Mathematics and Computation*, 218(16), 8005-8018.
20. Vita M.C., De Bartolo S., Fallico C., Veltri M. (2012) Usage of infinitesimals in the Menger’s Sponge model of porosity, *Applied Mathematics and Computation*, 218(16), 8187-8196.
21. Zhigljavsky A. (2012) Computing sums of conditionally convergent and divergent series using the concept of grossone, *Applied Mathematics and Computation*, 218(16), 8064-8076.
22. Sergeyev Ya.D. (2013) Solving ordinary differential equations by working with infinitesimals numerically on the Infinity Computer, *Applied Mathematics and Computation*, 219(22), 10668–10681.
23. Sergeyev Ya.D., Garro A. (2013) Single-tape and multi-tape Turing machines through the lens of the Grossone methodology, *Journal of Supercomputing*, 65(2), 645-663.
24. Sergeyev Ya.D. (2013) Numerical computations with infinite and infinitesimal numbers: Theory and applications, in “Dynamics of Information Systems: Algorithmic Approaches” edited by Sorokin, A., Pardalos, P.M., Springer, New York, pp. 1-66.

Invited plenary lectures and tutorials related to the topic of the lecture

- tutorial, International Conference and Summer School “Numerical Computations: Theory and Algorithms”, Falerna (CZ), Italy, June 17–23, 2013
- tutorial, international conference “Learning and Intelligent Optimization LION-7”, Catania, Italy, January 7-11, 2013
- keynote lecture opening the 4th International Joint Conference on Computational Intelligence, Barcelona, Spain, October 3-7, 2012
- invited plenary lecture and tutorial, 15’th GAMM-IMACS International Symposium on Scientific Computing, Computer Arithmetic and Verified Numerical Computations, Novosibirsk, Russia, September 24-28, 2012

- invited plenary lecture, the 4th International Conference on the Dynamics of Information Systems, February 20 - 22, 2012, Gainesville, Florida;
- invited lecture, the 12th International Symposium “Frontiers of Fundamental Physics”, Udine, Italy, November 21-23, 2011;
- invited plenary lecture, the 7th International Asian Summer School and Workshop “Optimization of Complex Systems”, Tashkent, Uzbekistan, October 17-27, 2011
- tutorial, Summer School “Achievements and Applications of Contemporary Informatics, Mathematics and Physics”, Kiev, Ukraine, August 12-19, 2011
- invited plenary lecture, the 4th International conference "Mathematics, its Applications and Mathematical Education" Russia, Ulan-Ude, Baikal, June 27 - July 1, 2011
- invited plenary lecture, the 15th Baikal International School-Seminar on Optimization Methods and their Applications, June 23–29, 2011
- invited 45 minutes lecture, the International Workshop “New Worlds of Computation”, Orléans, France, May 23-24, 2011
- invited one hour lecture, the Ninth International Conference on Cellular Automata for Research and Industry, Ascoli Piceno (Italy), September 21-24, 2010
- keynote lecture, the 2010 World Congress in Computer Science, Computer Engineering, and Applied Computing, Las Vegas, July 11-15, 2010
- tutorial, Infinite and Infinitesimal in Mathematics, Computing, and Natural Sciences, Cetraro, Italy, May 18-21, 2010
- invited plenary lecture, International conference "Optimization and Applications" (OPTIMA2009), Petrovac, Montenegro, September 21-25, 2009
- tutorial, the international workshop “Optimization and Control with Applications”, Harbin, China, June 6-11, 2009
- invited one hour lecture, the First World Congress on Global Optimization in Engineering & Science, Hunan, China, June 1-5, 2009
- invited one hour lecture, the International Workshop “New Worlds of Computation”, Orléans, France, January 12, 2009;
- Keynote address and a honorary plaque at the joint plenary session of the conferences WMSCI 2008, MEI 2008, EIC 2008, and BMIC 2008, Orlando, Florida, June, 29 - July, 2, 2008;
- invited one hour lecture, the World Congress of Nonlinear Analysts WCNA-2008, Orlando, Florida, July, 2-9, 2008;
- invited plenary lecture, International Conference of Numerical Analysis and Applied Mathematics ICNAAM 2007, September 16-20, 2007;
- invited lecture, the 8th Mediterranean Workshop and Topical Meeting "Novel Optical Materials and Applications", Cetraro, Italy, June 3-9, 2007;
- tutorial, the International Workshop on Learning and Intelligent Optimization, Andalo, Italy, February 12-18, 2007;
- invited one hour lecture, the 15th Euromicro Conference on Parallel, Distributed and Network-based Processing, Naples, February 7-9, 2007;
- invited plenary lecture opening the MATHESIS Congress, Trento, November 2-4, 2006;
- invited plenary lecture, the Third International Conference of Applied Mathematics, Plovdiv, Bulgaria, August 12-18, 2006;
- invited plenary lecture, International Conference “Applied Optimization and Metaheuristic Innovations”, Yalta, Ukraine, July 17-23, 2006;
- invited lecture at the second International Workshop on Variational Analysis and Partial Differential Equations, Erice (Sicily), under the auspices of the International School of Mathematics G. Stampacchia, July 5-14, 2006;
- round table “The Infinity Computer and Calculus”, the 8th Congress of SIMAI (La Societa' Italiana di Matematica Applicata e Industriale), Ragusa (Sicily), May 22-26, 2006;

- invited plenary lecture at the XXI Autumn Meeting of the Polish Information Processing Society, Katowice, Poland, December 5-9, 2005;
- invited plenary lecture at the International Workshop on Global Optimization, Almeria, Spain, September 18-22, 2005;
- tutorial at the International Conference on Complementarity, Duality, and Global Optimization in Science and Engineering, Blacksburg, Virginia, USA, August 15-17, 2005;
- invited plenary lecture, International Conference on Difference Equations, Special Functions and Applications, Munich, July 25-30, 2005;
- invited plenary lecture, International Conference “Numerical Analysis: the State of the Art”, Rende (CS), May, 19-21, 2005;
- invited semi-plenary lecture at the International conference on Selected Problems of Modern Mathematics, dedicated to the 200th anniversary of K.G. Jacobi and the 750th anniversary of the Koenigsberg foundation, Kaliningrad, April, 4-8, 2005;
- tutorial at the Workshop "Numerical Methods and Mathematical Software", Montecatini Terme (PT), January 31 - February 1, 2005;
- invited plenary lecture at the VI-th International Congress on Mathematical Modelling, Nizhni Novgorod, Russia, September, 21-26, 2004;
- opening invited lecture at the congress “The Infinity in Mathematics, Physics, and Philosophy”, Pisa, Italy, 26.03.2004.

Media citations

- Interview to the national Russian TV Channel “Culture”, 2013.
- Interview to ITAR-TASS, 7.11.2012.
- Article in MIT Technology Review, 19.03.2012.
- Article in The Voice of Russia, 10.02.2012.
- 45 minutes lecture at the national Russian TV Channel “Culture”, 2011.
- Article in the weekly magazine “Computerworld” (Russian Edition), #22, 2011.
- Article in TASS-TELECOM, August 11, 2011
- New Scientist (Russian Edition), # 3, 2011.
- Numerous articles have been dedicated to the Pythagoras International Prize for mathematics (the 1st Russian TV Channel, The Voice of Russia, ITAR-TASS, RIA Novosti, Rossiyskaya gazeta, Il Sole 24 Ore, Gazzetta del Sud, Il Quotidiano, Il Crotonese, Haberhurriyeti, etc.), 2010
- Article in Russian National newspaper “Ведомости” and e-newspaper ria.ru, 19.08.2010.
- Article in Italian e-newspaper “Gazzettino Europeo”, 26.06.2010.
- Article in Turkish newspaper “Haberhurriyeti”, 28.05.2010.
- Article in Italian newspaper “Calabria Ora”, 22.05.2010.
- Article in Italian newspaper “Il Quotidiano”, 18.05.2010.
- Article in Italian newspaper “Calabria Ora”, 18.05.2010.
- Article in Italian newspaper “Gazzetta del Sud”, 18.05.2010.
- Article in Italian monthly magazine “Newton”, March, 2010.
- Article in Russian monthly magazine “Наука и Жизнь” (Science and Life), 8, 2009.
- Article in Turkish newspaper “Haberhurriyeti”, 10.05.2009.
- Article in Italian quarterly magazine “Tutto Misura”, #2, 2008.
- Article in Italian monthly magazine “Quale Computer”, #12, 2007.
- Interview to Russian monthly magazine “Открытые системы”, #7, 2007.
- Two articles in Russian weekly magazine “Computerra”, #33, 2007.
- Article in the weekly magazine “Computerworld” (Russian Edition), #31, 2007.
- Article in Russian National e-newspaper “Наука.Известия”, 28.05.2007.

- Cover story in Italian National e-newspaper Key4biz: Knowledge for Business, 11.05.2007.
- Article in the leading Italian weekly magazine “Panorama”, 07.12.2006.
- Article in Russian National newspaper “Ведомости”, 30.11.2006.
- Book “Italian applications”, edited by F. Pedrocchi, 240 pages, Hublab Eds., 2006 and the corresponding site, describing the most promising Italian innovations.
- Interview to Italian radio station RadioCitta’Fujiko, 22.04.2006.
- Interview to the largest Italian National business newspaper “Il Sole 24 Ore”, 27.08.2005.
- Interview to Italian National newspaper “Italia Oggi” reprinted also by Italian National newspaper “MF” (Milan Finance), 12.07.2005.
- Interview to Italian National radio station Radio24, 23.06.2005.
- Cover story, department “Technology, Notable New Patents” of the U.S.A. magazine of intellectual property and technology “IpFrontline”, 15.06.2005.
- The Optimization Research Bridge Newsletter, Issue 9, March 2003.