

DETERMINISTIC GLOBAL OPTIMIZATION USING THE LIPSCHITZ CONDITION

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In this lecture, the global optimization problem of a multidimensional function satisfying the Lipschitz condition over a hyperinterval with an unknown Lipschitz constant is considered. It is supposed that the objective function can be “black box”, multiextremal, and non-differentiable. It is also assumed that evaluation of the objective function at a point is a time-consuming operation. Many algorithms for solving this problem have been discussed in literature. They can be distinguished, for example, by the way of obtaining information about the Lipschitz constant and by the strategy of exploration of the search domain.

Different exploration techniques based on various adaptive partition strategies are analyzed. The main attention is dedicated to diagonal algorithms, since they have a number of attractive theoretical properties and have proved to be efficient in solving applied problems. In these algorithms, the search hyperinterval is adaptively partitioned into smaller hyperintervals and the objective function is evaluated only at two vertices corresponding to the main diagonal of the generated hyperintervals.

It is demonstrated that the traditional diagonal partition strategies do not fulfil the requirements of computational efficiency because of executing many redundant evaluations of the objective function. A new adaptive diagonal partition strategy that allows one to avoid such computational redundancy is described. Some powerful multidimensional global optimization algorithms based on the new strategy are introduced. Results of extensive numerical experiments performed to test the methods proposed demonstrate their advantages with respect to diagonal algorithms in terms of both number of trials of the objective function and qualitative analysis of the search domain, which is characterized by the number of generated hyperintervals.

Selected references

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